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A NEW APPROACH FOR CRITERIA WEIGHT ELICITATION OF THE ARAS-H METHOD

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Abstract

Criteria weight inference is a crucial step for most of multi-criteria methods. However, criteria weights are often determined directly by the decision-maker (DM) which makes the results unreliable. Therefore, to overcome the imprecise weighting, we suggest the use of the preference programming technique. Instead of obtaining criteria weights directly from the DM, we infer them in a more objective manner to avoid the subjectivity and the unreliability of the results. Our aim is to elicit the ARAS-H criteria weights at each level of the hierarchy tree via mathematical programming, taking into account the DM's preferences. To put it differently, starting from preference information provided by the DM, we proceed to model our constraints. The ARAS-H method is an extension of the classical ARAS method for the case of hierarchically structured criteria. We adopt a bottom-up approach in order to elicit ARAS-H criteria weights, that is, we start by determining the elementary criteria weights (i.e. the criteria at the lowest level of the hierarchy tree). The solution of the linear programs is obtained using LINGO software. The main contribution of our criteria weight elicitation procedure is in overcoming imprecise weighting without excluding the DM from the decision making process.

Keywords: Multiple Criteria Decision Aiding, preference disaggregation, ARAS-H, criteria weights, mathematical programming.

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1 Introduction

Multiple criteria decision analysis (MCDA) is a general framework for supporting complex decision-making situations with multiple and often conflicting objectives (Belton and Stewart, 2002; Greco, Figueira and Ehrgott, 2016; Ishizaka and Nemery, 2013). Most of multi-criteria methods require fixing criteria weights in order to be implemented. Indeed, the problem of criteria weight determination has gained the interest of many researchers during the past decades. In fact, there are two types of weight elicitation: ‘a priori weights’, determined directly by experts, and ‘a posteriori weights’, obtained from the data (Jacquet-Lagrange and Siskos, 2001). This paper adopted the ‘a posteriori approach’.

We are therefore interested in reducing the subjectivity and the unreliability of weight values provided directly by the DM without excluding him from the decision making process. The paper is divided into five sections. In section 2, we present a brief state of the art survey of some weighting methods that deal with hierarchical structure of criteria. In section 3, we develop the criteria weight determination approach of the ARAS-H method. In section 4, an empirical example is presented to discuss the feasibility of the proposed model. In section 5, we present conclusions and our main perspectives.

2 A review of the literature

Very few authors have worked on criteria weight elicitation within hierarchical methods. To start with, Corrente, Greco and Slowiński (2016) proposed a generalization of the SRF (Simos-Roy-Figuiera) method (Figueira and Roy, 2002) to deal with weight elicitation in hierarchical structure of criteria. In the SRF method, the DM ranks the criteria from the least important to the most important with the possibility of ex-aequo between them. Then, he is asked to put some blank cards between two successive subsets of criteria to increase the difference of importance between the criteria in these two subsets. Finally, he defines the ratio z of the importance of the most important subset of criteria to that of the least important one. Moreover, Corrente et al. (2017) developed the imprecise SRF to deal with imprecise preference information given by the DM on the number of blank cards and on the ratio z . Therefore, the imprecise SRF method helps the DM to obtain the weights of criteria and sub-criteria on the basis of incompletely determined preference information. As a consequence of considering imprecise preference information in SRF, there is an infinity of compatible vectors of weights satisfying the constraints translating this preference information. Furthermore, Salo and Hämäläinen (1992) developed a preference programming method called Preference Assessment by Imprecise

Ratio Statements (PAIRS) in which the preference judgments are given as linear constraints on the weight ratios of the criteria and attribute-wise values of the alternatives. In addition, Keeney and Raiffa (1993) used Multi-Attribute Value Theory (MAVT) to elicit criteria weights. The attributes are grouped under more general upper level criteria and the weighting is carried out separately on each branch of the value tree. Thus, on each branch, the DM gives local weights to the criteria, which describe the relative importance of their consequence range under the ascending next level criterion. The overall weight of each attribute is calculated by multiplying its local weight by the local weights of all the ascending upper level criteria. On each branch of the value tree, the sum of the local weights is normalized to one. Consequently, the overall weight of each criterion is the sum of the overall weights of all its next level sub-criteria, and the sum of the overall weights of all the attributes will also be one. In what follows, we present an illustrative table of the criteria weights elicitation methods when dealing with a hierarchical structure.

Table 1: A review of methods of criteria weight determination

References	Criteria weight elicitation techniques	Methods
Corrente et al. (2017)	SRF	ELECTRE-III-H
Corrente, Greco and Slowiński (2016)	Extension of the SRF	ELECTRE Tri-H
Del Vasto-Terrientes et al. (2015a)	Simos	ELECTRE-III-H
Del Vasto-Terrientes et al. (2015b)	Simos	ELECTRE-III-H
Del Vasto-Terrientes et al. (2016a)	Simos	ELECTRE-TRI-B-H
Del Vasto-Terrientes et al. (2016b)	Subjective	ELECTRE-III-H

As can be seen in Table 1, the major studies used either the Simos' procedure or the SRF technique in order to elicit the hierarchical ELECTRE and PROMETHEE methods. However, both these methods have been criticized for their subjectivity. The Simos' method is based on an unrealistic assumption (lack of essential information) and leads to the process criteria having the same importance (i.e., the same weight) in a not robust way (Schärlig, 1996). Also, the Simos' and the SRF methods are considered to be subjective weighting ones. To overcome the imprecise weighting, we suggest preference programming which takes into account the DM's preferences. In an earlier paper, we suggested a criteria weight determination procedure for the ARAS method (Zavadskas and Turskis, 2010) through mathematical programming (Ghram and Frikha, 2018). Likewise, we proceed to develop a set of mathematical programs, which takes into account the DM's preferences, to elicit ARAS-H criteria weights at each node of the hierarchy tree. In fact, the ARAS-H method is an extension of the classical ARAS method in the case of hierarchically structured criteria (Ghram and Frikha, 2019; Ghram and Frikha, 2021).

3 The proposed model for ARAS-H criteria weight inference

The aggregation paradigm states that the aggregation model is known a priori, whereas the global preference is unknown. On the other hand, the philosophy of disaggregation involves the inference of preference models from the given global preferences. The development of preference disaggregation methods was initiated in 1978. In the disaggregation-aggregation approach, iterative interactive procedures are used to be aggregated later to a value system (Siskos, 1980). The first developed preference disaggregation methodology was the UTA (Jacquet-Lagrange and Siskos, 1982). The purpose of this method is to infer additive value functions from a given ranking through linear programming so that these functions are as consistent as possible with the global decision-maker's preferences. Thus, we adopt the preference disaggregation methodology in order to elicit criteria weights of the ARAS-H method. Our weight elicitation procedure is based on the solution of linear programs which take into account the DM's preferences. Consequently, the DM has to introduce some preference information which report his value system. Thus, this approach is based on preference relations provided by the decision maker, as well as on comparisons between differences of criteria weights and some weight partial pre-orders. By involving the DM in the weight elicitation process, we allow the DM to express his preference information not only comprehensively, but also partially, by considering preference information with respect to a sub-criterion at an intermediate level of the hierarchy. Thus, the DM can obtain results not only with respect to the comprehensive view, but also at the intermediate levels of the hierarchy. The process of weight elicitation is considered to be a set of mathematical programs. Their number depends on the number of the levels in the hierarchy. Henceforth, we adopt a bottom-up approach to elicit ARAS-H criteria weights. We start with the last level l . The aim is to obtain all elementary criteria weights from preference relations given by the DM on some pairs of alternatives according to intermediate criteria of the upward level. This process is generated until we reach the root criterion.

3.1 Determination of the elementary criteria weights

For each sub-criterion $G_{(r, n(r))}$, the DM is asked to give preference relations between some pairs of alternatives. Also, he is asked to provide some comparisons between differences of elementary criteria weights and certain elementary weight partial pre-orders. Those preference relations are modeled in Program 1. The solution of the following mathematical program will provide all elementary criteria weights.

Program 1:

The following notations have been introduced by Corrente, Greco and Slowiński (2012).

Let:

A be the set of alternatives;

EL : the set of indices of all elementary sub-criteria;

$n(r)$: the number of sub-criteria of G_r in the subsequent level;

$G_r \in G$, with $r = (i_1, \dots, i_h) \in I_G$, denote a sub-criterion of the first level criterion G_i at level h ;

$G_{(r, n(r))}$: the direct sub-criteria of G_r .

We define:

p to be the number of relations between a pair of alternative preferences provided by the DM;

z : a threshold;

w_j : the weight of the elementary criterion j ;

$\bar{x}_{Dj} / \bar{x}_{Fj}$ are the normalized performance values of the alternatives D and F , respectively, according to the elementary criterion j .

Thus, Program 1 can be written in the form:

$$\text{Max } \sum_{i=1}^p e_i \tag{1}$$

Subject to:

$$\sum_{j \in EL} w_j \bar{x}_{Dj} - \sum_{j \in EL} w_j \bar{x}_{Fj} - e_i \geq 0 \quad \forall D, F \in A; \forall i = 1, \dots, p \tag{2}$$

$$w_k - w_s \geq w_r - w_v \quad \forall k, s, r, v \in EL \tag{3}$$

$$w_k \geq w_l \quad \forall k, l \in EL \tag{4}$$

$$e_i \geq \frac{1}{2^{(p-1)}} \quad \forall i = 1, \dots, p \tag{5}$$

$$w_j \geq z \quad \forall j \in EL; z \geq 0 \tag{6}$$

$$\sum_{j \in EL} w_j = 1 \tag{7}$$

The objective function (eq. 1) expresses the maximization the sum of slack variables between a pair of alternatives as expressed by the DM. This slack variable insures a strict preference between two alternatives.

The first constraint concerns the degree of preference e_i , which measures the intensity of preference of an alternative over the other ones and is calculated as the difference between their utility degrees according to the intermediate sub-criterion $G_{(r, n(r))}$. In other words, in the ARAS-H method, all alternatives are ranked according to a decreasing order of their utility degree values. For instance, an alternative D is preferable to F is equivalent to: the utility degree of D is greater than that of F on intermediate sub-criterion $G_{(r, n(r))}$.

Then, $K_{G(r,n(r))}(D) \geq K_{G(r,n(r))}(F)$.

Consequently, $\frac{S_D}{S_0} \geq \frac{S_F}{S_0}$ on intermediate sub-criterion $G_{(r,n(r))}$, where S_0 is the best value.

Equally, $\sum_{j \in EL} \hat{x}_{Dj} \geq \sum_{j \in EL} \hat{x}_{Fj}$,

where \widehat{x}_{Dj} and \widehat{x}_{Fj} are the weighted normalized values of all elementary criteria.

Signify, $\sum_{j \in EL} w_j \bar{x}_{Dj} \geq \sum_{j \in EL} w_j \bar{x}_{Fj}$,

with \bar{x}_{Dj} and \bar{x}_{Fj} being the normalized values of the decision making matrix.

Next, the preference relations expressed by the DM are modeled in the mathematical program as: $\sum_{j \in EL} w_j \bar{x}_{Dj} - \sum_{j \in EL} w_j \bar{x}_{Fj} - e_i \geq 0 \quad \forall D, F \in A; \forall i = 1, \dots, p$ (eq. 2).

Besides the preference relations, the DM must provide two other information types. The first one concerns comparisons of the differences of adjacent weights presented as: $w_k - w_s \geq w_r - w_v$ (eq. 3). Therefore, the gap between the importance of elementary criteria k and l is more important than that between r and v . The second information type concerns a partial pre-order on elementary criteria weights. Nevertheless, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l \quad \forall k, l \in EL$ (eq. 4). The number of partial pre-order constraints must not exceed $(n - 1)$. In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, we must impose that all slack variables (e_i) are strictly positive. Consequently, we have to fix a minimum threshold associated with each e_i related to each preference relation. It is evident that the threshold value is strongly dependent on the number of preference relationships. As an illustration, the threshold value is fixed to be $\frac{1}{2^{(p-1)}}$. We introduce the constraint $e_i \geq \frac{1}{2^{(p-1)}} \quad \forall i=1, \dots, p$ (eq. 5) into the mathematical Program 1. The constraint (eq. 6) concerns the threshold of the weight values. Indeed, in the constraints of the weight determination mathematical program, we must take into account the requirement that all criteria weights must be strictly positive ($w_j > 0$) in order to restrict any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j . Therefore, we must add to the mathematical program the constraint $w_j \geq z \quad \forall j \in EL$ (eq. 6). In addition, we must take into consideration that criteria weights are normalized. It means that the sum of weights of elementary criteria must be equal to 1 ($\sum_{j \in EL} w_j = 1$; eq. 7).

3.2 Determination of the intermediate sub-criteria weights

We note that if the number of the levels in the hierarchy exceeds three ($l > 3$), Program 2 is used. It is repeated until we reach the first level of intermediate criteria. Consequently, the DM is asked to give the same information as in Program 1, but this time according to the first-level intermediate criteria G_i . Those preference relations are included in Program 2. Hence, the solution of Program 2 gives the weights of the intermediate criteria at level h .

Program 2:

Corrente, Greco and Slowiński (2012) have introduced the following notations:

I_G : the set of indices of the particular criteria representing the positions of the criteria in the hierarchy;

$G_r \in G$, with $r = (i_l, \dots, i_h) \in I_G$: a sub-criterion of the first-level criterion G_i at level h ;

$LB(G_r)$: the set of indices of sub-criteria of the second-last level descending from criterion/sub-criterion G_r .

Thus, Program 2 can be written in the form:

$$\text{Max } \sum_{q=p}^t e_q \tag{8}$$

Subject to:

$$\sum_{j \in LB(G_r)} w_j K_j(D) - \sum_{j \in LB(G_r)} w_j K_j(F) - e_q \geq 0 \quad \forall D, F \in A; q = p, \dots, t \tag{9}$$

$$w_k - w_s \geq w_r - w_v \quad \forall k, s, r, v \in LB(G_r) \tag{10}$$

$$w_k \geq w_l \quad \forall k, l \in LB(G_r) \tag{11}$$

$$e_q \geq \frac{1}{2^{(p-1)}} \quad \forall q = p, \dots, t \tag{12}$$

$$w_j \geq z \quad \forall j \in LB(G_r); z \geq 0 \tag{13}$$

$$\sum_{j \in LB(G_r)} w_j = 1 \tag{14}$$

Likewise, we have to maximize in the objective function (eq. 8) of Program 2, the sum of slack variables e_q , to insure the strict preference and to avoid indifference between two alternatives.

The constraint (eq. 9) concerns the degree of preference e_q , which measures the intensity of preference of an alternative over the other ones and is calculated as the difference between their utility degrees according to the first-level criterion G_{ih} . In the ARAS-H method, the statement that an alternative D is preferable over alternative F ($D \succ F$) on the first-level criterion G_{ih} is expressed by $K_{G_{ih}}(D) \geq K_{G_{ih}}(F)$. Therefore, $\sum_{j \in LB(G_r)} w_j K_j(D) \geq \sum_{j \in LB(G_r)} w_j K_j(F)$.

In addition to the preference relations, the DM must provide some comparisons of the differences of adjacent weights in the form: $w_k - w_s \geq w_r - w_v$ (eq. 10). Therefore, the gap between the importance of intermediate criteria k and l is more important than that between r and v . Also, the DM is asked to give a partial pre-order on intermediate criteria weights. Furthermore, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l \forall k, l \in LB(G_r)$ (eq. 11). However, the number of partial pre-order constraints must not exceed $(n - 1)$.

In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, strictly positive slack variables (e_q) are imposed. Consequently, we have to fix a minimum threshold associated with each e_q , related to each preference relation equals to $\frac{1}{2^{(p-1)}}$. Thus, $e_q \geq \frac{1}{2^{(p-1)}} \forall q = 1, \dots, p$ (eq. 12).

The constraint (17) concerns the threshold of the weight values. Indeed, we must take into account the fact that all criteria weights must be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j . Next, we must add to the mathematical program the constraint $w_j \geq z \forall j \in LB(G_r)$ (eq. 13). In addition, we must take into account that criteria weights are normalized, that is, the sum of weights of intermediate criteria descending from G_{ih} must be equal to 1 ($\sum_{j \in LB(G_r)} w_j = 1$; eq. 14).

3.3 Determination of the first-level intermediate criteria weights

The DM is asked to give a preference relation between a pair of alternatives according to the *root criterion*. He is also asked to provide some comparisons between differences of the first-level intermediate criteria weights and some first-level intermediate weight partial pre-orders. The solution of Program 3 gives the weights of the first-level intermediate criteria.

Program 3:

Corrente, Greco and Slowiński (2012) have defined the following notations:
 m : the number of the first-level criteria (root criteria) $G_1 \dots G_m$;
 I_G : the set of indices of particular criteria representing the position of the criteria in the hierarchy.

Hence, Program 3 can be written in the form:

$$\text{Max } e_{t+1} \tag{15}$$

Subject to:

$$\sum_{j \in I_G} w_j K_j(D) - \sum_{j \in I_G} w_j K_j(F) - e_{t+1} \geq 0 \quad \forall D, F \in A \quad (16)$$

$$w_k - w_s \geq w_r - w_v \quad \forall k, s, r, v \in I_G \quad (17)$$

$$w_k \geq w_l \quad \forall k, l \in I_G \quad (18)$$

$$e_{t+1} \geq \frac{1}{2^t} \quad (19)$$

$$w_j \geq z \quad \forall j \in I_G; z \geq 0 \quad (20)$$

$$\sum_{j \in I_G} w_j = 1 \text{ for the root criterion } G_m. \quad (21)$$

In order to insure strict preference and to avoid indifference between two alternatives, we have to maximize in the objective function of Program 3, the slack variable e_{t+1} (eq. 15). As we said before, in the ARAS-H method, the statement that an alternative D is preferable than alternative F ($D > F$) on the root criterion G_m is expressed by $K_{G_m}(D) \geq K_{G_m}(F)$, that is, $\sum_{j \in I_G} w_j K_j(D) \geq \sum_{j \in I_G} w_j K_j(F)$. In fact, e_{t+1} presents the degree of preference of D over F and it is interpreted as the difference between the two utility degrees according to the root criterion G_m (eq. 16). As an illustration, $K_{I_G}(D) - K_{I_G}(F) = e_{t+1}$ for the preference relation $(t + 1)$ provided by the DM. In addition to the preference relation, the DM must provide two other information types. The first one concerns comparisons of the differences of adjacent weights in the form: $w_k - w_s \geq w_r - w_v$ (eq. 17). Therefore, the gap between the importance of the first-level intermediate criteria k and l is more important than that between r and v . The second information type concerns a partial pre-order on the first-level intermediate criteria weights. Nevertheless, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l \quad \forall k, l \in I_G$ (eq. 18). The number of partial pre-order constraints must not exceed $(n - 1)$. In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, we must impose that the slack variable (e_{t+1}) be strictly positive. Consequently, we have to fix a minimum threshold associated with the slack variable (e_{t+1}). Thus, we introduce the constraint $e_{t+1} \geq \frac{1}{2^t}$ (eq. 19).

The constraint (20) concerns the threshold of the weight values. Surely, all criteria weights must be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j . Henceforth, we must add to the mathematical program the constraint $w_j \geq z \quad \forall j \in I_G$. In addition, we must take into consideration that criteria weights are normalized. It means that the sum of weights of the first-level intermediate

criteria descending from the root criterion G_m must be equal to 1. For instance, if we have m root criteria, then $\sum_{j \in I_G} w_j = 1$ (eq. 21).

Our approach is iterative interactive. Within the iterative process of determining ARAS-H criteria weights, the DM can add or remove information whenever he is not satisfied with the given result. The additional information consists in adding or withdrawing one or more preference relations. Thus, each preference relation is modeled in a mathematical program as a constraint. Certainly, in real-world decision problems, decision-makers have difficulties with providing reliable information due to time restriction and their cognitive limitations. The preferences of decision makers are therefore not necessarily stable: they can contain inconsistent and conflicting data. The role of an interactive tool is to help the DM to understand his preferences and their representation in a multi-criteria aggregation method. Inconsistencies appear when DM's preferences cannot be guaranteed by the aggregation method used.

4 An illustrative example

The aim of this example is to present websites designed to promote tourist destination brands. Websites have become crucial tools for communicating destination brands and for selling a variety of tourism services and related products (Fernández-Cavia and Huertas-Roig, 2010). Since the problem is considered to be a complex one, the DM organized the set of criteria into a hierarchical structure as expressed in Figure 1. Thus, ten tourist websites in different regions in the world (Andalusia, Catalonia, Barcelona, Madrid, Santiago de Compostela, Rias Baixas, Stockholm, Wales, Rome and Switzerland) are evaluated, according to a set of hierarchically structured criteria. The following dataset comes from a Spanish research project entitled “Online Communication for Destination Brands. Development of an Integrated Assessment Tool: Websites, Mobile Applications and Social Media (CODETUR)” completed in 2012, whose main objective was to identify a website evaluation framework to help expert managers to enhance and optimize online communication of their brands. In this section, we discuss the analysis of this dataset with the criteria weight elicitation procedure of the ARAS-H method.

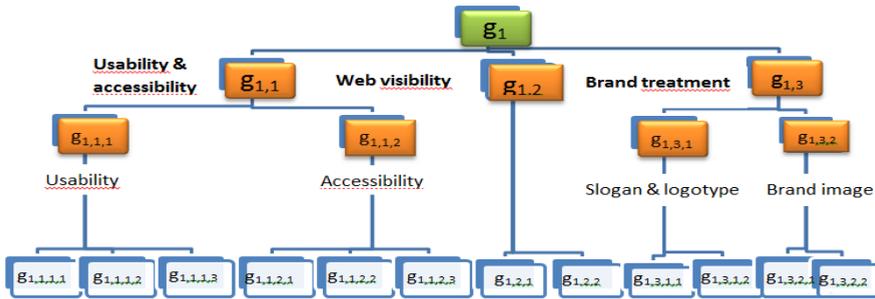


Figure 1: Criteria hierarchy tree

Table 2: Normalised decision matrix

	$g_{1.1.1.1}$	$g_{1.1.1.2}$	$g_{1.1.1.3}$	$g_{1.1.2.1}$	$g_{1.1.2.2}$	$g_{1.1.2.3}$	$g_{1.2.1}$	$g_{1.2.2}$	$g_{1.3.1.1}$	$g_{1.3.1.2}$	$g_{1.3.2.1}$	$g_{1.3.2.2}$
Andalusia	1	1	0.6	0	0.125	0.16	0.58	0.42	1	0.5	1	0.43
Catalonia	0.48	0	0	0.67	0.375	0.387	0	0	0	1	0.4	0
Barcelona	1	0.43	0.8	0.67	0.125	1	0.61	0.86	0	0.9	1	1
Madrid	0.66	1	0.6	0.33	0.5	0.613	0.64	0.44	0	0	0	0.43
Santiago	1	0.71	0.2	0.33	0	0.32	1	0.31	1	0.25	0.6	0.43
Rias Baixas	0	0	0	0.67	0.125	0.16	0.19	0	1	0	0	0
Stockholm	1	1	1	1	0.125	0.387	0.61	0.75	1	0.4	0.2	0.57
Wales	0.66	0.57	0	1	0.375	0	0.89	0.22	1	0.5	0.6	0
Rome	1	1	1	1	0.125	0.16	0.61	0.22	0	0	0.4	0.43
Switzerland	0.52	1	1	1	0.125	0.16	0.75	1	1	0.75	1	1

The DM is asked to give his preference relations at each node of the hierarchy. On Usability ($g_{1.1.1}$): the DM prefers Madrid over Wales
 Mad > Wales means,
 $K_{(1.1.1)}(\text{Mad}) - K_{(1.1.1)}(\text{Wales}) - e_1 \geq 0$. Then,
 $(w_{(1.1.1.1)} \bar{x}_{(1.1.1.1)}(\text{Mad}) + w_{(1.1.1.2)} \bar{x}_{(1.1.1.2)}(\text{Mad}) + w_{(1.1.1.3)} \bar{x}_{(1.1.1.3)}(\text{Mad})) -$
 $- ((w_{(1.1.1.1)} \bar{x}_{(1.1.1.1)}(\text{Wales}) + w_{(1.1.1.2)} \bar{x}_{(1.1.1.2)}(\text{Wales}) + w_{(1.1.1.3)} \bar{x}_{(1.1.1.3)}(\text{Wales})) - e_1 \geq 0$

On Accessibility ($g_{1.1.2}$): the DM prefers Stockholm over Rias
 Stock > Rias is equivalent to:
 $K_{(1.1.2)}(\text{Stock}) - K_{(1.1.2)}(\text{Rias}) - e_2 \geq 0$. In other words,
 $(w_{(1.1.2.1)} \bar{x}_{(1.1.2.1)}(\text{Stock}) + w_{(1.1.2.2)} \bar{x}_{(1.1.2.2)}(\text{Stock}) + w_{(1.1.2.3)} \bar{x}_{(1.1.2.3)}(\text{Stock})) -$
 $- ((w_{(1.1.2.1)} \bar{x}_{(1.1.2.1)}(\text{Rias}) + w_{(1.1.2.2)} \bar{x}_{(1.1.2.2)}(\text{Rias}) + w_{(1.1.2.3)} \bar{x}_{(1.1.2.3)}(\text{Rias})) - e_2 \geq 0$

On Web visibility ($g_{1.2}$): the DM prefers Switzerland over Andalusia
 Switz > Anda means,
 $K_{(1.2)}(\text{Switz}) - K_{(1.2)}(\text{Anda}) - e_3 \geq 0$. Consequently,

$$(w_{(1,2,1)} \bar{x}_{(1,2,1)} (\text{Switz}) + w_{(1,2,2)} \bar{x}_{(1,2,2)} (\text{Switz})) - (w_{(1,2,1)} \bar{x}_{(1,2,1)} (\text{Anda}) + w_{(1,2,2)} \bar{x}_{(1,2,2)} (\text{Anda})) - e_3 \geq 0$$

On Slogon & Logotype ($g_{1,3,1}$): the DM prefers Santiago over Rome

Sant \succ Rome Thus,

$$K_{(1,3,1)} (\text{Sant}) - K_{(1,3,1)} (\text{Rome}) - e_4 \geq 0. \text{ In other terms,}$$

$$(w_{(1,3,1,1)} \bar{x}_{(1,3,1,1)} (\text{Sant}) + w_{(1,3,1,2)} \bar{x}_{(1,3,1,2)} (\text{Sant})) - (w_{(1,3,1,1)} \bar{x}_{(1,3,1,1)} (\text{Rome}) + w_{(1,3,1,2)} \bar{x}_{(1,3,1,2)} (\text{Rome})) - e_4 \geq 0$$

On Brand Image ($g_{1,3,2}$): the DM prefers Barcelona over Wales

Barc \succ Wales means that:

$$K_{(1,3,2)} (\text{Barc}) - K_{(1,3,2)} (\text{Wales}) - e_5 \geq 0. \text{ Hence,}$$

$$(w_{(1,3,2,1)} \bar{x}_{(1,3,2,1)} (\text{Barc}) + w_{(1,3,2,2)} \bar{x}_{(1,3,2,2)} (\text{Barc})) - (w_{(1,3,2,1)} \bar{x}_{(1,3,2,1)} (\text{Wales}) + w_{(1,3,2,2)} \bar{x}_{(1,3,2,2)} (\text{Wales})) - e_5 \geq 0$$

Hence, the DM provides other information types concerning the thresholds of both weights and slack variables, in addition to comparisons between differences of elementary criteria weights and some elementary criteria weights partial pre-orders.

We use the LINGO software for the solution of the three mathematical programs.

Program 1:

$$\text{Max } \sum_{i=1}^5 e_i$$

Subject to:

$$w_{1112} \times 0.43 + w_{1113} \times 0.6 - e_1 \geq 0$$

$$w_{1121} \times 0.33 + w_{1123} \times 0.23 - e_2 \geq 0$$

$$w_{121} \times 0.17 + w_{122} \times 0.58 - e_3 \geq 0$$

$$w_{1311} + w_{1312} \times 0.25 - e_4 \geq 0$$

$$w_{1321} \times 0.4 + w_{1322} - e_5 \geq 0$$

$$w_{1321} - w_{1311} \geq w_{1312} - w_{1322}$$

$$w_{1123} - w_{1121} \geq w_{1122} - w_{1112}$$

$$w_{122} - w_{1112} \geq w_{1111} - w_{121}$$

$$w_{1111} \geq w_{1121}$$

$$w_{1122} \geq w_{1311}$$

$$w_{1321} \geq w_{1113}$$

$$w_{1312} \geq w_{1322}$$

$$w_{1111} + w_{1112} + w_{1113} + w_{1121} + w_{1122} + w_{1123} + w_{121} + w_{122} + w_{1311} + w_{1312} + w_{1321} + w_{1322} = 1$$

$$e_i \geq 0.0625 \quad i = 1 \dots 5$$

$$w_j \geq 0.015 \quad j \in \{(1,1,1,1), (1,1,1,2), (1,1,1,3), (1,1,2,1), (1,1,2,2), (1,1,2,3), (1,2,1), (1,2,2), (1,3,1,1), (1,3,1,2), (1,3,2,1), (1,3,2,2)\}$$

The solution of Program 1 provided the elementary criteria weights.

W1112	0.3902952E-01
W1113	0.7619551E-01
W1121	0.1500000E-01
W1123	0.2502174
W121	0.1500000E-01
W122	0.1033621
W1311	0.1500000E-01
W1312	0.1900000
W1321	0.7619551E-01
W1322	0.1900000
W1122	0.1500000E-01
W1111	0.1500000E-01

After determining all elementary criteria weights, we proceed to the construction of the weighted-normalized decision matrix (see Table 3 in Appendix).

Thus, the obtained utility degrees values K_i will be used in constraints of type: $\sum_{j \in LB(G_r)} w_j K_j(B) - \sum_{j \in LB(G_r)} w_j K_j(Q) - e_i \geq 0 ; \forall B, Q \in A; \forall i = p, \dots, t$ (see Table 4 as an example in Appendix).

On Usability and Accessibility ($g_{1,1}$): the DM prefers Stockholm over Catalonia Stock > Cata means,

$$K_{(1,1)}(\text{Stock}) - K_{(1,1)}(\text{Cata}) - e_6 \geq 0. \text{ Hence, } (w_{(1,1,1)} K_{(1,1,1)}(\text{Stock}) + w_{(1,1,2)} K_{(1,1,2)}(\text{Stock})) - (w_{(1,1,1)} K_{(1,1,1)}(\text{Cata}) + w_{(1,1,2)} K_{(1,1,2)}(\text{Cata})) - e_6 \geq 0$$

On Brand treatment ($g_{1,3}$): the DM prefers Andalusia over Santiago Anda > Sant is equivalent to,

$$K_{(1,3)}(\text{Anda}) - K_{(1,3)}(\text{Sant}) - e_7 \geq 0. \text{ Thus, } (w_{(1,3,1)} K_{(1,3,1)}(\text{Anda}) + w_{(1,3,2)} K_{(1,3,2)}(\text{Anda})) - (w_{(1,3,1)} K_{(1,3,1)}(\text{Sant}) + w_{(1,3,2)} K_{(1,3,2)}(\text{Sant})) - e_7 \geq 0$$

Similarly, the DM provides other information types concerning the thresholds of both weights and slack variables, in addition to comparisons between differences of intermediate criteria weights and some intermediate criteria weights partial pre-orders. Thus, Program 2 can be written in the form:

Program 2:

$$\begin{aligned} & \text{Max } \sum_{q=6}^7 e_q \\ & w_{111} \times 0.856 - w_{112} \times 0.27 - e_6 \geq 0 \\ & w_{131} \times 0.217 + w_{132} \times 0.2 - e_7 \geq 0 \\ & w_{111} \geq w_{131} \end{aligned}$$

$$w_{132} \geq w_{112}$$

$$w_{132} - w_{111} \geq w_{112} - w_{131}$$

$$w_{132} - w_{131} \geq w_{111} - w_{112}$$

$$w_{111} + w_{112} + w_{131} + w_{132} = 1$$

$$e_q \geq 0.0625 \quad \forall q = 6, 7$$

$$w_j \geq 0.015 \quad j \in \{(1,1,1), (1,1,2), (1,3,1), (1,3,2)\}$$

The solution of Program 2 gave us the weights of $g_{1,1,1}$, $g_{1,1,2}$, $g_{1,3,1}$ and $g_{1,3,2}$.

W111	0.4850000
W112	0.1500000E-01
W131	0.1500000E-01
W132	0.4850000

The final step in the weight elicitation process is to calculate the values of the utility degree K_i with respect to the first-level sub-criteria using the previously obtained weights (see Table in Appendix as an example).

On the root criterion g_1 : the DM prefers Rome over Madrid

Rome $>$ Mad means,

$K_1(\text{Rome}) - K_1(\text{Mad}) - e_8 \geq 0$. Therefore,

$$(w_{(1,1)} K_{(1,1)}(\text{Rome}) + w_{(1,2)} K_{(1,2)}(\text{Rome}) + w_{(1,3)} K_{(1,3)}(\text{Rome})) -$$

$$(w_{(1,1)} K_{(1,1)}(\text{Mad}) + w_{(1,2)} K_{(1,2)}(\text{Mad}) + w_{(1,3)} K_{(1,3)}(\text{Mad})) - e_8 \geq 0$$

In the same way, the DM provide us with other information type concerning the thresholds of both weights and slack variables in addition to comparisons between differences of first-level intermediate criteria weights and some first-level intermediate criteria weight partial pre-orders. Therefore, Program 3 can be written in the form:

Program 3:

Max e_8

$$w_{11} \times 0.0778 - w_{12} \times 0.2099 + w_{13} \times 0.097 - e_8 \geq 0$$

$$w_{11} - w_{13} \geq w_{13} - w_{12}$$

$$w_{11} \geq w_{13}$$

$$w_{11} + w_{12} + w_{13} = 1$$

$$e_8 \geq 0.008$$

$$w_j \geq 0.015 \quad j \in \{(1,1), (1,2), (1,3)\}$$

The solution of Program 3 gave us the weights of the first-level intermediate criteria to construct the complete pre-order (Figure 2) from ranking the utility degrees K_i obtained in Table 7 (see Appendix).

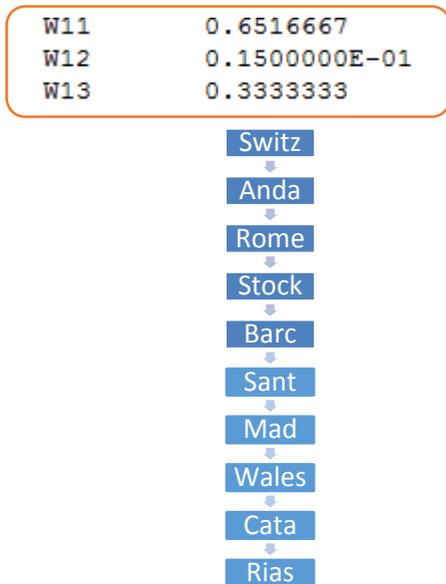


Figure 2: The complete pre-order

As we can notice, Switzerland outranks all the other alternatives. It is considered to be the best web tourist destination brand, whereas Rias Baixas is considered to be the worst.

In the final analysis, the proposed model can be summarized in the following Figure 3.

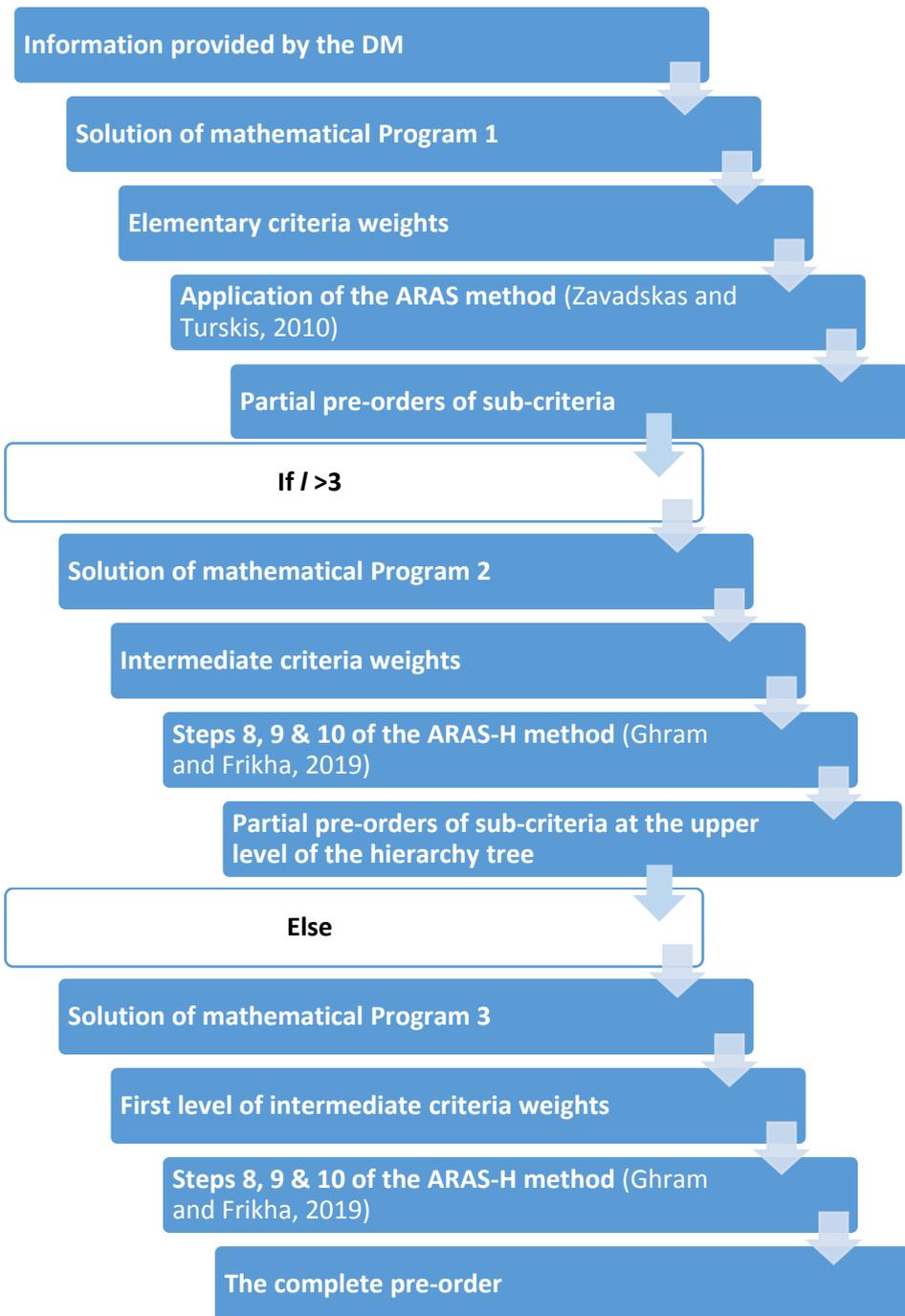


Figure 3: A flow chart of the proposed approach

5 Conclusion and perspectives

In this paper, we developed a criteria weight elicitation procedure for the ARAS-H method. Its aim is to overcome the imprecise weighting encountered in most of multi-criteria aggregation problems, in which the DM determines directly the weight values using his own intuition. However, the direct weight elicitation is too subjective, which makes the results unreliable. To overcome this issue, we suggested a weighting method based on preference programming which takes into account the DM's preferences. Therefore, the DM is involved indirectly in the decision-making process by expressing his preference relations on some pairs of alternatives, some comparisons between differences of criteria weights and some weight partial pre-orders. A set of mathematical programs were developed and solved by the LINGO software package in order to elicit ARAS-H criteria weights at each level of the hierarchy tree. Therefore, the DM can express his preference information not only in a comprehensive way, but also in a partial way, that is, considering preference information with respect to each criterion in the hierarchy tree. Thus, he can analyze the obtained rankings according to each criterion apart from detecting the main anomalies of the given problem. An illustrative example was presented at the end of the paper to showcase the feasibility of the proposed approach by ranking the tourist destination brands across Europe. The main contribution of this paper is that the DM is not involved directly in the weight elicitation, which reduces the subjectivity of the results. Thus, he interacts partially through preferential information. Nevertheless, the proposed model is valid only for the ARAS-H method. For future research, we consider developing the ARAS-H method in the context of a fuzzy environment and to elicit criteria weights of the fuzzy ARAS-H method.

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Appendix

Table 3: Weighted normalized decision matrix

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$	$g_{1,3,2,2}$
Andalusia	0.039	0.076	0.009	0	0.001875	0.01648	0.0087	0.0798	0.076	0.095	0.015	0.00645
Catalonia	0.01872	0	0	0.1675	0.005625	0.0398301	0	0	0	0.19	0.006	0
Barcelona	0.039	0.03268	0.012	0.1675	0.001875	0.103	0.00915	0.1634	0	0.171	0.015	0.015
Madrid	0.02574	0.076	0.009	0.0825	0.0075	0.0631699	0.0096	0.0836	0	0	0	0.00645
Santiago	0.039	0.05396	0.003	0.0825	0	0.03296	0.015	0.0589	0.076	0.0475	0.009	0.00645
Rias Baixas	0	0	0	0.1675	0.001875	0.01648	0.00285	0	0.076	0	0	0
Stockholm	0.039	0.076	0.015	0.25	0.001875	0.0398301	0.00915	0.1425	0.076	0.076	0.003	0.00855
Wales	0.02574	0.04332	0	0.25	0.005625	0	0.01335	0.0418	0.076	0.095	0.009	0
Rome	0.039	0.076	0.015	0.25	0.001875	0.01648	0.00915	0.0418	0	0	0.006	0.00645
Switzerland	0.02028	0.076	0.015	0.25	0.001875	0.01648	0.01125	0.19	0.076	0.1425	0.015	0.015

Table 4: Optimality values and utility degrees of the alternatives according to sub-criterion «Usability»

$g_{1,1,1}$	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	S_i	K_i	Rank
Andalusia	0.039	0.076	0.009	0.124	0.953846154	2
Catalonia	0.01872	0	0	0.01872	0.144	8
Barcelona	0.039	0.03268	0.012	0.08368	0.643692308	6
Madrid	0.02574	0.076	0.009	0.11074	0.851846154	4
Santiago	0.039	0.05396	0.003	0.09596	0.738153846	5
Rias Baixas	0	0	0	0	0	9
Stockholm	0.039	0.076	0.015	0.13	1	1
Wales	0.02574	0.04332	0	0.06906	0.531230769	7
Rome	0.039	0.076	0.015	0.13	1	1
Switzerland	0.02028	0.076	0.015	0.11128	0.856	3

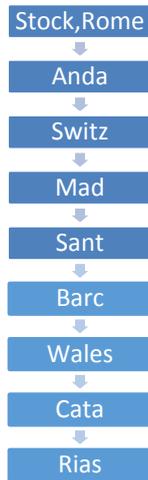


Figure 4: Partial pre-order according to sub-criterion «Usability»

Table 5: Optimality values and utility degrees of the alternatives according to sub-criterion «Accessibility»

$g_{1.1.2}$	$g_{1.1.2.1}$	$g_{1.1.2.2}$	$g_{1.1.2.3}$	S_i	K_i	Rank
Andalusia	0	0.065	0.0528	0.1178	0.230128	9
Catalonia	0.1005	0.195	0.127611	0.423111	0.826568	3
Barcelona	0.1005	0.065	0.33	0.4955	0.967983	2
Madrid	0.0495	0.26	0.202389	0.511889	1	1
Santiago	0.0495	0	0.1056	0.1551	0.302995	8
Rias Baixas	0.1005	0.065	0.0528	0.2183	0.42646	7
Stockholm	0.15	0.065	0.127611	0.342611	0.669307	5
Wales	0.15	0.195	0	0.345	0.673974	4
Rome	0.15	0.065	0.0528	0.2678	0.52316	6
Switzerland	0.15	0.065	0.0528	0.2678	0.52316	6

Table 6: Utility degrees of the alternatives according to the first-level sub-criterion «Usability & Accessibility»

$g_{1.1}$	K_i
Andalusia	0.463
Catalonia	0.081
Barcelona	0.326
Madrid	0.421
Santiago	0.364
Rias Baixas	0.009
Stockholm	0.5
Wales	0.271
Rome	0.499
Switzerland	0.429



Figure 5: Partial pre-order according to the first-level sub-criterion «Usability & Accessibility»

Table 7: Utility degrees and the ranking of the alternatives according to the «Root criterion»

g_i	K_i	Rank
Andalusia	0.4197	2
Catalonia	0.0888	9
Barcelona	0.3761	5
Madrid	0.3082	7
Santiago	0.322	6
Rias Baixas	0.008	10
Stockholm	0.3902	4
Wales	0.2281	8
Rome	0.3908	3
Switzerland	0.444	1