

Sebastian Sitarz^{*}
Krzysztof Botor^{**}

MOLPTOL – A SOFTWARE PACKAGE FOR SENSITIVITY ANALYSIS IN MOLP

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Abstract

The paper introduces a new software package, MOLPTOL, for sensitivity analysis in multi-objective linear programming. In this application, which is available for free of charge on the web page (<https://sites.google.com/view/molptol>), the tolerance approach as a measure of sensitivity is used. The motivation for creating MOLPTOL is the lack of such tools to date. MOLPTOL is novel for multi-criteria decision-making methods based on sensitivity analysis. The paper presents some new computational methods for obtaining the supremal tolerances as well.

Keywords: multi-objective linear programming, sensitivity analysis, computer software.

1 Introduction

The general idea of using sensitivity analysis in optimization aims to deal with the uncertainty and imprecise data of the considered model. Sensitivity analysis plays an important role in decision problems as well. Usually it is used in the case of perturbations of parameters which often appears in real-life problems. In this paper we consider sensitivity analysis in multiple-objective linear programming (MOLP) problems. Since many constraints and objectives are formulated in a linear way, MOLP problems are often used in practice. Here, we focus on maintaining efficiency of a given efficient solution taking into account the perturbation that can be applied simultaneously to objective functions coefficients.

* Independent Researcher, e-mail: sitarzseba@gmail.com, ORCID: 0000-0002-1757-1390.

** BPSC Sp. z o.o., al. Różdzieńskiego 188H, 40-203 Katowice, Poland, e-mail: krzysiekbotor@gmail.com.

The main drawback of sensitivity analysis in MOLP is the lack of tools in multi-criteria decision making problems based on sensitivity analysis. In general, such tools should be easily applied by a decision maker. Hence, software which helps to analyze these problems would be a significant simplification of the decision process. Moreover, the computational methods to obtain the measures of sensitivity (for example, the supremal tolerances) need development. Due to the above disadvantages in this field, this paper provides new computational methods of sensitivity analysis and shows their properties. Based on these methods, MOLPTOL – a new software package for obtaining supremal tolerances in MOLP problems – is presented. Moreover, MOLPTOL can be also used as a tool in decision problems in which sensitivity analysis is important for the decision maker. In such problems the perturbation of objective functions coefficients is taken into account. The solution proposed by MOLPTOL can be easily used on the dedicated web page: <https://sites.google.com/view/molptol>.

The paper consists of the following sections: section 1 provides the introduction; section 2 presents related papers related to the subject of the paper; section 3 introduces the basic objects and notation; section 4 presents the theoretical background of computation methods used in MOLPTOL; section 5 shows the MOLPTOL software; section 6 describes a market model; section 7 illustrates an application of MOLPTOL in the market model; and the final section summarizes the paper.

2 Related papers

The following approaches to sensitivity analysis in MOLP are worth mentioning:

- the tolerance approach,
- the range set approach,
- the standard approach,
- the robust approach,
- the partial preference relations approach.

Let us shortly describe the above approaches by presenting the related papers.

The tolerance approach aims to find a value (tolerance) representing the perturbation that can be applied simultaneously to objective functions coefficients without affecting the efficiency of a given efficient solution. We distinguish two main forms of the tolerance approach: additive and percentage tolerances. In the case of the additive tolerance we focus on additive perturbations. The percentage tolerance approach, however, represents the relative (percentage) perturbations. This approach in linear programming comes from Wendell (1982). The use of the tolerance approach in MOLP was proposed

by Hansen, Labbe and Wendell (1989). Hladik (2008a, 2008b) develops this concept in theoretical and computational ways, while Borges and Antunes (2002) present sensitivity analysis of the weights in MOLP. Applications of the tolerance approach in the transportation problem can be found in the papers by Paratane and Bit (2020), as well as Badra (2004, 2006).

The range set approach of to sensitivity analysis comes from linear programming theory for sensitivity analysis of optimal solutions, see Gass (1975) and Gal (1995). The use of this approach in MOLP was proposed by Benson (1985). The range set approach aims to find the values of parameter that can be applied to a given direction of the objective coefficients without affecting the efficiency. Methods of computing the range set in MOLP are given by Hladik et al. (2019).

The standard approach to sensitivity analysis is the extension of this method in linear programming (Gal, 1995). Initial research on using this approach to MOLP was done by Sitarz (2010, 2011). The standard approach aims to find values (a parameter set) of one selected objective function coefficient that can be applied without affecting the efficiency. Pourkarimi (2015) proposes building a ranking of all efficient faces by using stability measures based on standard sensitivity analysis.

The robust approach presented by Georgiev, Luc and Pardalos (2013) consists of analysis of the efficient solutions that remain efficient when the objective matrix is slightly perturbed by means of the Euclidean norm for the matrix of objective functions coefficients. Moreover, in that paper we find algorithms to compute the radius of robustness. In turn, Pourkarimi and Soleimani-Damaneh (2016) propose the so-called robustness order which is defined as the interiority order of the matrix of objective functions coefficients. In the paper by Goberna et al. (2015), MOLP problems with uncertainty both in the objective function and the constraints are considered.

The partial preference relations introduced by Podinovski (2012) present sensitivity analysis in the form of a parametric partial order. This approach can be applied to the sensitivity analysis by taking into account the changes of parameters of the order. Moreover, Podinovski and Potapov (2019) expand this theory by introducing parameters connected with boundaries of intervals for criteria value tradeoffs uncertainty.

3 MOLP problem and tolerance approach

In this paper, we consider the following MOLP problem:

$$VMax \{Cx: x \in X\}, \tag{1}$$

where $X = \{x \in \mathbb{R}^n: Ax \leq b\} \subset \mathbb{R}^n$ is a given set, with $A \in \mathbb{R}^{m,n}$ and $b \in \mathbb{R}^m$; matrix $C \in \mathbb{R}^{k,n}$ is given by the linear objective functions $c^i x$ for $i = 1, \dots, n$. One can find a detailed description of MOLP problems in books by Steuer (1986) or Zeleny (1982). A feasible solution $x^* \in X$ is called an efficient solution to (1) if there is no $x \in X$ such that:

$$Cx^* \leq Cx \wedge Cx^* \neq Cx.$$

We can check efficiency of the given feasible solution by using the following theorem, Ehrgott (2005).

Theorem 1. A feasible solution x^* is efficient if and only if the following linear program:

$$\begin{aligned} & \text{Max } e^T w \\ & Cx - Iw = Cx^* \\ & x \in X \\ & w \geq 0, \end{aligned}$$

where I is identity matrix and e is vector of ones, has an optimal objective function value of zero.

We consider the sensitivity in the sense of the remaining efficiency of a given feasible solution $x^* \in X$. Furthermore, we analyze the sensitivity analysis in the case of changing matrix C . Let matrix $G \in \mathbb{R}^{n,k}$ be given. We introduce a δ, G -neighbourhood of matrix $C = [c_{ij}]$ as follows:

$$O_{\delta,G}(C) = \{D = [d_{ij}] \in \mathbb{R}^{n,k}: |d_{ij} - c_{ij}| < \delta |g_{ij}| \text{ if } g_{ij} \neq 0, d_{ij} = c_{ij} \text{ if } g_{ij} = 0\}.$$

In this case we consider the following problem obtained from (1) by using $D \in O_{\delta,G}(C)$:

$$V\text{Max } \{Dx: x \in X\}. \tag{2}$$

Definition 1. A tolerance for an efficient solution x^* is any real δ such that x^* remains efficient to (2) for all $D \in O_{\delta,G}(C)$. The supremal tolerance is denoted by δ^{sup} .

We look closer at the two types of the tolerance presented above: an additive tolerance and percentage tolerance. These tolerances represent the additive perturbation and the percentage perturbation of all coefficients of matrix C .

Definition 2. An additive tolerance is a tolerance for matrix G consisting only of ones:

$$g_{ij} = 1 \text{ for all } i, j.$$

Definition 3. A percentage tolerance is a tolerance for matrix G consisting of $|c_{ij}|$:

$$g_{ij} = |c_{ij}| \text{ for all } i, j.$$

4 Computation methods used in MOLPTOL

Computing the supremal tolerance is based on problem (3) given by Hladik and Sitarz (2013):

$$\begin{aligned} \delta^{sup} &= \text{Min } \delta \\ A^1(x - x^*) &\leq 0 \\ C(x - x^*) + \delta G|x - x^*| &\geq 0 \\ e^T G|x - x^*| &= 1 \\ \delta &\geq 0, \end{aligned} \quad (3)$$

where A^1 is a submatrix of A consisting only of the active constraints for x^* and e is a vector of ones. Moreover, G is a given matrix representing the method of perturbation of the coefficients of matrix C (the way of introducing matrix G was presented in section 3).

Problem (3) is NP hard, thus we are looking to improve it. In MOLPTOL, we can improve computation by using the properties of (3) and two methods: decomposition procedure and bisection procedure. The detailed descriptions of the above methods are given in the next subsections.

4.1 Decomposition procedure

Computing the supremal tolerance by using the decomposition method is based on the decomposition of problem (3) into 2^n simpler problems, according to the signs of $(x - x^*)_i$. The composition is given by a vector $z \in \{\pm 1\}^n$. For each vector z we build the following problem:

$$\begin{aligned} \delta_z &= \text{Min } \delta \\ A^1(x - x^*) &\leq 0 \\ C(x - x^*) + \delta G \text{diag}(z)(x - x^*) &\geq 0 \\ \text{diag}(z)(x - x^*) &\geq 0 \\ e^T G \text{diag}(z)(x - x^*) &= 1 \\ \delta &\geq 0, \end{aligned} \quad (4)$$

where matrix $\text{diag}(z)$ is a diagonal matrix with the coefficients z_i .

The supremal tolerance is given by the following equation:

$$\delta^{sup} = \text{Min}_{z \in \{\pm 1\}^n} \delta_z.$$

Problem (4), proposed by Hladik and Sitarz (2013), is not NP-hard, but it is still difficult to solve. Thus, we propose the following method which is based on solving the sequence of linear programming problems to obtain δ^{sup} .

Theorem 2. The feasible set of problem (4) with $\delta = \bar{\delta}$ is non-empty if and only if $\delta_z \leq \bar{\delta}$.

Proof. Let us assume that the feasible set of problem (4) for $\delta = \bar{\delta}$ is non-empty. In this case, there exists a pair $(\bar{\delta}, \bar{x})$ which is a feasible solution for (4). Since δ_z is the minimum of all feasible δ , we have $\delta_z \leq \bar{\delta}$.

Now, let us assume that $\delta_z \leq \bar{\delta}$. Hence, δ_z is an optimal solution for (4); it is also a feasible solution for (4) with some x_z . Moreover, by using the inequality:

$$diag(z)(x_z - x^*) \geq 0,$$

we obtain:

$$0 \leq C(x_z - x^*) + \delta_z Gdiag(z)(x_z - x^*) \leq C(x_z - x^*) + \bar{\delta} Gdiag(z)(x_z - x^*).$$

Thus, the pair $(\bar{\delta}, x_z)$ fulfils the above condition of (4). Furthermore, the rest of the conditions of (4) are fulfilled as well. Thus $(\bar{\delta}, x_z)$ is a feasible solution for (4).

Corollary 1. The feasible set of problem (4) is empty with $\delta = \bar{\delta}$ if and only if $\delta_z > \bar{\delta}$.

We use the following theorem to check if the feasible set of problem (4) is non-empty.

Theorem 3. The feasible set for (4) with $\delta = \bar{\delta}$ is non-empty if and only if the following linear problem:

$$\begin{aligned} &Min \ v \\ &A^1(x - x^*) \leq 0 \\ &-C(x - x^*) - \bar{\delta} Gdiag(z)(x - x^*) \leq 0 \\ &\quad -diag(z)(x - x^*) \leq 0 \\ &e^T Gdiag(z)(x - x^*) + v = 1 \\ &\quad v \geq 0, \end{aligned} \tag{5}$$

has an optimal objective function value of zero.

Proof. By substituting $\delta = \bar{\delta}$ into problem (4) and introducing a new non-negative variable v , we obtain (after some operations) the following linear constraints for the feasible set of (4):

$$\begin{aligned}
 A^1(x - x^*) &\leq 0 \\
 -C(x - x^*) - \bar{\delta}Gdiag(z)(x - x^*) &\leq 0 \\
 -diag(z)(x - x^*) &\leq 0 \\
 e^T Gdiag(z)(x - x^*) + v &= 1.
 \end{aligned}$$

By using Proposition 6.15 from Ehrgott (2005), the above linear set of constraints is non-empty if and only if problem (5) has an optimal objective function value of zero.

Remark 1. The main property of problem (5) is that it is a linear programming problem, thus it is easy to solve.

4.2 Sets of vectors $z \in \{\pm 1\}^n$

Let Z denote the set of all vectors $z \in \{\pm 1\}^n$. The number of elements of set Z is very important: it can reduce the computation time. Thus, we focus on Z more closely. In this subsection we present methods to narrow Z . First, let us present some observations (Hladik and Sitarz, 2013):

(i) If condition $x \geq 0$ is assumed, then for i such that $x_i^* = 0$, we set $z_i = 1$ (in other words we omit the case of $z_i = -1$).

By setting $y = diag(z)(x - x^*)$ we obtain:

(ii) Let $i \in \{1, \dots, n\}$. If $\min_y \{y_i : A^1 y \leq 0\} \geq 0$, then we set $z_i = 1$ (we omit $z_i = -1$).

(iii) Let $i \in \{1, \dots, n\}$. If $\max_y \{y_i : A^1 y \leq 0\} \leq 0$, then we set $z_i = -1$ (we omit $z_i = 1$).

Applying (i), (ii) and (iii), we obtain the initial set $Z_0 \subseteq Z$, for which:

$$\delta^{sup} = \text{Min}_{z \in Z} \delta_z = \text{Min}_{z \in Z_0} \delta_z. \tag{6}$$

Now, we proceed with a new observation used in our method which is based on problem (5) and theorems 2 and 3. By taking into account this new observation we can omit some vectors z in formula (6). Suppose that we have a set $Z_i \subseteq Z$ and:

$$\text{Min}_{z \in Z} \delta_z = \text{Min}_{z \in Z_i} \delta_z. \tag{7}$$

Moreover, let $\bar{\delta}$ be given (which is the approximate value of δ^{sup}).

Definition 4. Let \bar{Z}_i be defined as follows:

$$\bar{Z}_i = \{z \in Z_i : \text{problem (5) has an optimal objective function value of zero with } z \text{ and } \bar{\delta}\}.$$

By using definition 4, we formulate the next theorem.

Theorem 4. If $\bar{Z}_i \neq \emptyset$, then:

$$\delta^{sup} \leq \bar{\delta} \text{ and } \text{Min}_{z \in Z_i} \delta_z = \text{Min}_{z \in \bar{Z}_i} \delta_z.$$

Otherwise, $\bar{\delta} \leq \delta^{sup}$.

Proof. If $\bar{Z}_i \neq \emptyset$, then there exists \bar{z} such that problem (5) has an optimal objective function value of zero with $\bar{\delta}$. Thus, by using theorems 2 and 3, $\delta_{\bar{z}} \leq \bar{\delta}$. Moreover, by using the fact that:

$$\delta^{sup} = \text{Min}_{z \in \{\pm 1\}^n} \delta_z,$$

we have $\delta^{sup} \leq \delta_{\bar{z}} \leq \bar{\delta}$, which means that $\delta^{sup} \leq \bar{\delta}$. Moreover, for all $z \notin \bar{Z}_i$ we have (theorem 2) $\bar{\delta} < \delta_z$, which means that $\delta_{\bar{z}} < \delta_z$, thus:

$$\text{Min}_{z \in \bar{Z}_i} \delta_z \leq \delta_{\bar{z}} \leq \text{Min}_{z \notin \bar{Z}_i} \delta_z.$$

Since $Z_i = \bar{Z}_i \cup \bar{Z}'_i$ we have:

$$\text{Min}_{z \in Z_i} \delta_z = \text{Min}_{z \in \bar{Z}_i} \delta_z.$$

If $\bar{Z}_i = \emptyset$, then for all $z \in Z_i$ we have (theorem 2) $\bar{\delta} < \delta_z$, thus by using (7) we obtain $\bar{\delta} \leq \delta^{sup}$.

Remark 2. The main property of theorem 4 is the fact that it is possible to reduce set Z in order to find δ^{sup} . We omit vectors z for which problem (5) does not have an optimal objective function value of zero.

4.3 An algorithm for obtaining δ^{sup}

In this algorithm we use the bisection procedure for seeking the supremal tolerance. While the idea of bisection is taken from the optimization numerical methods, in the case of supremal tolerance, the bisection has been adopted together with theorem 4. Figure 1 presents the algorithm to obtain δ^{sup} . Let us introduce the parameters and their initial values:

i – index for steps,

Z_i – the set of vectors z considered in step i ; we start with Z_0 defined in subsection 4.2,

δ^L – the left endpoint of interval; we have the initial constraint $\delta^{sup} \geq 0$; moreover, in most cases, the supremal tolerance is close to zero; thus, the initial value of δ^L is equal to zero,

δ^R – the right endpoint of interval; according to our numerical experiments, the initial value of δ^R should be taken as follows: $\delta^R = \max_{i,j} |c_{ij}|$,

$\bar{\delta}$ – the middle-point of interval; from the nature of the bisection method the initial value of $\bar{\delta}$ is equal to $(\delta^L + \delta^R)/2$,

S – number of steps; the precision of the approximate value of δ^{sup} is related to the number of steps; to have the error equal to $\varepsilon = 0.001$, we should take $S = \lceil \log_2(\delta^R/\varepsilon) \rceil + 1$.

Step $i \geq 0$: If $i = S$, then the approximate value of δ^{sup} is equal to $\bar{\delta}$, otherwise proceed as follows:

Check if $\bar{Z}_i \neq \emptyset$

If yes, then set:

$\delta^R = \bar{\delta}$

$\bar{\delta} = (\delta^L + \delta^R)/2$

$i = i + 1$

$Z_i = \bar{Z}_{i-1}$

and go to Step i

If not, then set:

$\delta^L = \bar{\delta}$

$\bar{\delta} = (\delta^L + \delta^R)/2$

$i = i + 1$

$Z_i = Z_{i-1}$

and go to Step i

Figure 1: Algorithm to obtain δ^{sup}

5 MOLPTOL – a short description of the software

MOLPTOL is a software package that runs on Windows systems with the .NET 4.0 platform installed. It handles the problem in the form of (1). Moreover, the non-negativity condition ($x \geq 0$) can be added by one click. The sensitivity analysis of a given vector x^* proceeds by means of the two approaches: the supremal additive tolerance and the supremal percentage tolerance. Moreover, MOLPTOL uses Express, a numerical tool (free of charge) that is a version of the Microsoft Solver Foundation library (MSF). A description of this library can be found on the web page: <http://msdn.microsoft.com>. The software can be used free of charge on the web page: <https://sites.google.com/view/molptol>.

6 A market model

In economic theory, there is a market model studied in isolation (Mas-Colell, Whinston and Green, 1995). We consider a model with N goods and M agents. The initial endowment of agent i is given by vector $e_i = (e_{1,i}, \dots, e_{N,i})$. Let u_i denote the linear utility function of agent i . Each agent wants to maximize his utility function. The feasible allocations are the vectors

$x = (x_{1,1}, \dots, x_{N,1}, \dots, x_{1,M}, \dots, x_{N,M}) \geq 0$ which for all $i \in \{1, \dots, N\}$ satisfy the following condition:

$$\sum_{j=1}^M x_{i,j} = \sum_{j=1}^M e_{i,j}$$

By using the above description, we can formulate the following MOLP problem:

$$\begin{aligned} &VM\max \begin{bmatrix} u_1(x_{1,1}, \dots, x_{N,1}), \\ \dots \\ u_M(x_{1,M}, \dots, x_{N,M}) \end{bmatrix} \\ &\sum_{j=1}^M x_{i,j} = \sum_{j=1}^M e_{i,j}, \text{ for } i \in \{1, \dots, N\} \\ &x_{1,1}, \dots, x_{N,1}, \dots, x_{1,M}, \dots, x_{N,M} \geq 0. \end{aligned} \tag{8}$$

The efficient solutions of problem (8) are called the Pareto optimal allocations. The decision problem in the market model is to find an allocation which is Pareto optimal and satisfies additional decision maker's preferences.

7 An application of MOLPTOL in the market model

The analysis of the market model can be done by means of sensitivity analysis of the initial data, which may be imprecise and changeable, especially the coefficients of the utility functions. We look for the Pareto allocations which are the least sensitive by means of changing these coefficients. Thus, by using MOLPTOL, we check if the given allocations are Pareto optimal. Moreover, we compute the supremal tolerances for these allocations. For further analysis, we recommend the allocation with the biggest supremal tolerance. We proceed to such an analysis by using MOLPTOL in the following case scenario. We consider a model with three goods, three agents, and the following initial endowments:

$$e_1 = (2,4,2), \quad e_2 = (2,2,2), \quad e_3 = (6,2,2).$$

Moreover, the agents have the following utility functions:

$$u_1(x_{1,1}, x_{2,1}, x_{3,1}) = x_{1,1} + 4x_{2,1} + 5x_{3,1},$$

$$u_2(x_{1,2}, x_{2,2}, x_{3,2}) = x_{1,2} + x_{2,2},$$

$$u_3(x_{1,3}, x_{2,3}, x_{3,3}) = x_{2,3} + x_{3,3}.$$

The MOLP problem connected with the above market model takes the following form:

$$\begin{aligned}
 & VMax \begin{bmatrix} x_{1,1} + 4x_{2,1} + 5x_{3,1} \\ x_{1,2} + x_{2,2} \\ x_{2,3} + x_{3,3} \end{bmatrix} \\
 & x_{1,1} + x_{2,1} + x_{3,1} = 10 \\
 & x_{1,2} + x_{2,2} + x_{3,2} = 8 \\
 & x_{1,3} + x_{2,3} + x_{3,3} = 6 \\
 & x_{1,1}, x_{2,1}, x_{3,1}, x_{1,2}, x_{2,2}, x_{3,2}, x_{1,3}, x_{2,3}, x_{3,3} \geq 0.
 \end{aligned} \tag{9}$$

The initial parameters for MOLPTOL are as follows:

$$C = \begin{bmatrix} 1 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 10 \\ 8 \\ 6 \end{bmatrix}.$$

Moreover, the non-negativity condition is assumed. Let us analyze the following allocations:

$$\begin{aligned}
 x^a &= (2, 4, 2, 8, 2, 0, 0, 2, 4), \\
 x^b &= (0, 0, 6, 10, 0, 0, 0, 8, 0), \\
 x^c &= (5, 0, 0, 5, 0, 0, 0, 8, 6).
 \end{aligned}$$

By using MOLPTOL we obtain that:

- allocation x^a is not efficient,
- allocation x^b is efficient and the supremal percentage tolerance is equal to 5.57%,
- allocation x^c is efficient and the supremal percentage tolerance is equal to 99.98%.

Thus, for further consideration, allocation x^a is omitted (because is not efficient). Furthermore, we conclude that allocation x^b is more sensitive than allocation x^c (based on the values of the supremal percentage tolerance). From this point of view, allocation x^c is better than allocation x^b . The consideration above includes only sensitivity analysis; in an actual decision-making problem, more aspects should be taken into consideration. However, the presented analysis can help make a decision in which sensitivity analysis is important.

8 Summary

Sensitivity analysis in MOLP problems by means of the tolerance approach was considered. New computational methods to obtain the supremal tolerances were provided as well. The methods used the decomposition procedure and the bisection procedure. Based on the proposed algorithm, MOLPTOL – a new software package for obtaining supremal tolerances in MOLP problems – was presented. It can be used free of charge on the web page: <https://sites.google.com/view/molptol>. A market model and an application of MOLPTOL to it were presented. The application illustrated the possibilities of using MOLPTOL in decision problems in which sensitivity analysis is important for the decision maker. Further research and improvement of MOLPTOL will consist of:

- adding other sensitivity analysis methods, for instance, standard sensitivity analysis or the range set approach;
- extending the software by introducing fuzzy numbers or interval coefficients;
- taking into account other tools beside MSF, for instance, the Gurobi solver;
- finding more applications of MOLPTOL.

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